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SEPARATION OF A BINARY MIXTURE IN A THERMODIFFUSION COLUMN WITH A SPIRAL WINDING

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The solution of the transfer equation in a thermodiffusion column with spiral winding in sampling conditions is given for the case $c(1-c) \approx$ const.

In [1], the transfer equation for a cylindrical column with a spiral winding (Fig. 1) was obtained*

$$\frac{\partial c}{\partial \theta} = (\cos^2\varphi + k)\frac{\partial^2 c}{\partial u^2} + 2\sin\varphi\cos\varphi\frac{\partial^2 c}{\partial u \partial v} + (\sin^2\varphi + k)\frac{\partial^2 c}{\partial v^2} - \cos\varphi\frac{\partial}{\partial u}[c(1-c)] - \sin\varphi\frac{\partial}{\partial v}[c(1-c)] - \varkappa\frac{\partial c}{\partial u}, \quad (1)$$

where

$$u = \frac{H^{(0)}z}{K_c^{(0)}},\quad v = \frac{H^{(0)}y}{K_c^{(0)}},\quad \theta = \frac{H^{(0)2}t}{\rho\delta K_c^{(0)}},\quad k = \frac{K_d^{(0)}}{K_c^{(0)}},\quad \varkappa = \frac{\sigma}{BH^{(0)}}. \quad (2)$$

In the present work, a solution is given to the problem of separating a binary liquid mixture for which the condition $c(1-c) \approx$ const $= a$ is observed throughout the whole process, and the coefficient k satisfies the inequalities

$$k \ll \cos^2\varphi;\ k \ll \sin^2\varphi. \quad (3)$$

Since in the separation of liquid mixtures k is of the order of $5 \cdot 10^{-3}$, it follows from Eq. (3) that

$$15^\circ \leqslant \varphi \leqslant 75^\circ. \quad (4)$$

Then in the steady state Eq. (1) is replaced by

$$\cos^2\varphi\frac{\partial^2 c}{\partial u^2} + 2\sin\varphi\cos\varphi\frac{\partial^2 c}{\partial u \partial v} + \sin^2\varphi\frac{\partial^2 c}{\partial v^2} - \varkappa\frac{\partial c}{\partial u} = 0. \quad (5)$$

The solution of Eq. (5) must satisfy the following conditions.

1. The flux along the z axis — see Eqs. (2.7) and (2.9) of [1] — is determined by the expression

$$j_z^* = \frac{H^{(0)}}{\delta}c(1-c)\cos\varphi - \frac{K_c^{(0)}}{\delta}\cos^2\varphi\frac{\partial c}{\partial z} - \frac{K_c^{(0)}}{\delta}\sin\varphi\cos\varphi\frac{\partial c}{\partial y} + \frac{\sigma}{B\delta}c.$$

This flux at the outlet from the apparatus (when z = L) should be equal to $\sigma c_e/B\delta$, which, taking account of the first two expressions in Eq. (2), leads to the result

$$\left(a - \cos\varphi\frac{\partial c}{\partial u} - \sin\varphi\frac{\partial c}{\partial v}\right)_{u=u_e} = 0,$$

*In [1], the errors corrected in [2, 3] were still present in Eq. (32).

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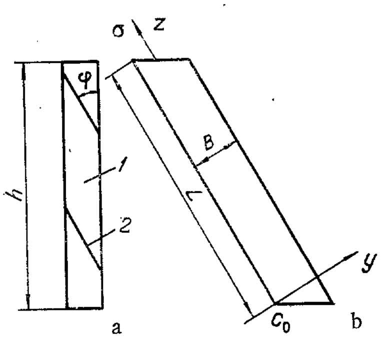

Fig. 1. Thermodiffusion column 1 with spiral winding 2 (a) and view of unwound spiral tape (b).

which after introducing the new variable

$$w = c - c_0 \tag{6}$$

takes the form

$$\left(a - \cos\varphi \frac{\partial w}{\partial u} - \sin\varphi \frac{\partial w}{\partial v} \right)_{u=u_e} = 0. \tag{7}$$

2. It is required that, at the input to the spiral, i.e., when u = 0 and v = 0, the concentration of the separable mixture is c_0 or w = 0.

The new independent variables

$$\xi = u;\ \eta = \frac{1}{\varkappa}(u - v \operatorname{ctg} \varphi) \tag{8}$$

are introduced, when Eq. (5) may be replaced by

$$\cos^2\varphi \frac{\partial^2 w}{\partial \xi^2} - \varkappa \frac{\partial w}{\partial \xi} = \frac{\partial w}{\partial \eta}, \tag{9}$$

and Eq. (7) by

$$\left(a - \cos\varphi \frac{\partial w}{\partial \xi} \right)_{\xi=u_e} = 0. \tag{10}$$

Equation (9) is the heat-conduction equation, which may be solved by the Laplace–Karson method of integral transformations when $\eta \geqslant 0$ or

$$u \geqslant v \operatorname{ctg} \varphi. \tag{11}$$

The second condition leads to the expression

$$w|_{\eta=0} = 0. \tag{12}$$

After reverting to the initial variables, the solution of Eq. (9), taking Eqs. (10) and (12) into account, may be written in the form

$$w = \frac{a\cos\varphi}{u_e} \exp\left[-\frac{\varkappa}{2\cos^2\varphi}(u_e - u) \right] \left[1 - \exp\left(-\frac{\varkappa u}{\cos^2\varphi} \right) \right] + \\ + \frac{2a\cos\varphi}{u_e} \exp\left[-\frac{\varkappa}{2\cos^2\varphi}(u_e - u) \right] \sum_{n=1}^{\infty} \frac{\mu_n \sin\mu_n \frac{u}{u_e} \exp[-a_n(u - v\operatorname{ctg}\varphi)]}{\varkappa a_n (2u_e a_n + 1)\cos\mu_n}, \tag{13}$$

where

$$a_n = \frac{1}{\varkappa}\frac{\mu_n^2}{u_e^2}\cos^2\varphi + \frac{\varkappa^2}{4\cos^2\varphi}. \tag{14}$$

Direct substitution of Eq. (13) into Eqs. (5) and (7) indicates that they are satisfied.

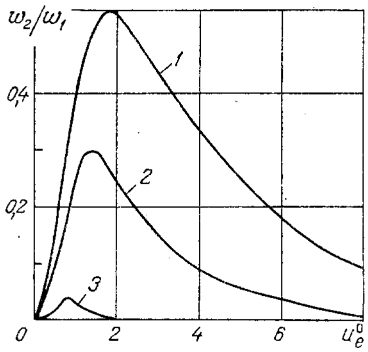

Fig. 2. Dependence of the ratio of the second term in Eq. (19) to the first on the dimensionless height of the column with different angles of slope of the spiral: 1) 15; 2) 30; 3) 45°; $v_e^{(0)}/u_e^{(0)}=0.2$; $\varkappa_0=1$; $u_e^{(0)}=1$.

The quantities μ_n appearing in Eq. (13) are the roots of the equation

$$\operatorname{tg}\mu_n = -2\frac{\cos^2\varphi}{\varkappa u_e}\mu_n. \tag{15}$$

For the whole column, i.e., when $u = u_e$, taking account of Eq. (15), it is found that

$$w_e = \frac{a\cos\varphi}{\varkappa}\left[1-\exp\left(-\frac{\varkappa u_e}{\cos^2\varphi}\right)\right] - \frac{4a\cos^3\varphi}{\varkappa u_e^2}\sum_{n=1}^{\infty}\frac{\mu_n^2}{\varkappa a_n(2u_e a_n+1)}\exp[-a_n(u_e - v\operatorname{ctg}\varphi)]. \tag{16}$$

Note that, when $u = u_e$, Eq. (13) also satisfies the condition that the fluxes over the coordinate y are equal to zero at the boundaries of the spiral winding, i.e., when $v = 0$ and $v = v_e$; these fluxes, as shown in [1], take the form

$$j_y^* = \frac{H^{(0)}}{\delta}\sin\varphi\left(a - \sin\varphi\frac{\partial w}{\partial v} - \cos\varphi\frac{\partial w}{\partial u}\right).$$

The mean value of w over the coordinate v

$$w_{e,\mathrm{m}} = \frac{1}{v_e}\int_0^{v_e} w\,dv,$$

taking account of Eq. (16), takes the form

$$w_{e,\mathrm{m}} = \frac{a\cos\varphi}{\varkappa}\left[1-\exp\left(-\frac{\varkappa u_e}{\cos^2\varphi}\right)\right] + \frac{4a\cos^2\varphi\sin\varphi}{v_e u_e^2}\sum_{n=1}^{\infty}\frac{\mu_n^2}{\varkappa^2 a_n^2(2u_e a_n+1)}\times$$
$$\times\left\{\exp\left[-a_n u_e\left(1-\frac{v_e}{u_e}\operatorname{ctg}\varphi\right)\right] - \exp(-a_n u_e)\right\}. \tag{17}$$

It is taken into account that in a cylindrical column with a spiral winding

$$u_e = \frac{u_e^{(0)}}{\cos\varphi};\quad v_e = v_e^{(0)}\cos\varphi;\quad \varkappa = \frac{\varkappa_0}{\cos\varphi}, \tag{18}$$

where

$$u_e^{(0)} = \frac{H^{(0)}h}{K_c^{(0)}},\quad v_e^{(0)} = \frac{H^{(0)}B_0}{K_c^{(0)}},\quad \varkappa_0 = \frac{\sigma}{B_0 H^{(0)}}. \tag{18a}$$

Substituting Eq. (18) into Eq. (17) gives

$$w_{e,\mathrm{m}} = w_1 + w_2 = \frac{a\cos^2\varphi}{\varkappa_0}\left[1-\exp\left(-\frac{\varkappa_0 u_e^{(0)}}{\cos^4\varphi}\right)\right] + \frac{4a\cos^3\varphi\sin\varphi}{\varkappa_0^2 v_e^{(0)}}\sum_{n=1}^{\infty}\frac{\mu_n^{(0)2}}{b_n^2(2b_n+1)}\times$$
$$\times\left\{\exp\left[-b_n\left(1-\frac{v_e^{(0)}}{u_e^{(0)}}\frac{\cos^3\varphi}{\sin\varphi}\right)\right] - \exp(-b_n)\right\}, \tag{19}$$

TABLE 1. Values of the First Root of Eq. (21)

$\varkappa_0 u_e^{(0)}$	φ, deg				
	15	30	45	60	75
0,25	1,65722	1,70072	1,83660	2,28890	3,03310
0,50	1,73500	1,81140	2,02870	2,57040	3,08625
1,00	1,86896	1,99070	2,28890	2,80440	3,11365
2,00	2,07617	2,24130	2,57040	2,95870	3,12755
4,00	2,34579	2,52420	2,80440	3,04667	3,13456
8,00	2,62291	2,77012	2,95873	3,09330	3,138072

where

$$b_n = \frac{u_e^{(0)}}{\varkappa_0}\left(\frac{\mu_n^{(0)2}}{u_e^{(0)2}}\cos^4\varphi + \frac{\varkappa_0^2}{4\cos^4\varphi}\right). \tag{20}$$

The first term w_1 in Eq. (19) corresponds to the solution obtained when transfer along the y axis is neglected. This solution was given in [4]. The second term w_2 gives the contribution introduced by taking account of transfer along the y axis.

Taking account of Eq. (18), Eq. (15) takes the form

$$\operatorname{tg}\mu_n^{(0)} = -2\frac{\cos^4\varphi}{\varkappa_0 u_e^{(0)}}\mu_n^{(0)}. \tag{21}$$

Calculations show that the second and subsequent terms of the series in the second term in Eq. (19) are negligibly small in comparison with the first, for which the values of the roots are given in Table 1.

It is evident from Eqs. (19) and (20) that the increase in concentration of the target component depends on four quantities: φ, $u_e^{(0)}$, $\varkappa_0$, and $v_e^{(0)}/u_e^{(0)}$; according to Eq. (18a), the last of these is the ratio of the column perimeter to its height. With increase in the ratio $v_e^{(0)}/u_e^{(0)}$, w_2 increases; however, there is a constraint here following from Eq. (11), which, together with Eq. (4), leads to the limiting value $v_e^{(0)}/u_e^{(0)} < 0.288$.

An illustration of this contribution, which is introduced into the increase in concentration by the second term of Eq. (19), is given in Fig. 2, from which it is evident that the greatest value of w_2 in comparison with w_1 is reached at $\varphi = 15°$, while the contribution of the second term decreases significantly with increase in the angle. As shown by the calculations, $w_2 \ll w_1$ when $u_e^{(0)} < 0.5$, and only the first term need be retained in Eq. (19). In this case, the results coincide with those of [4].

NOTATION

c, mass concentration; $u_e = H^{(0)}L/K_c^{(0)}$; $v_e = H^{(0)}B/K_c^{(0)}$, $H^{(0)} = \alpha\rho^2 g\beta\delta^3(\Delta T)^2/6!\eta\overline{T}$, $K_c^{(0)} = g^2\rho^3\beta^2\delta^7(\Delta T)^2/9!\eta^2 D$, $K_d^{(0)} = \rho D\delta$; z, y, coordinates (see Fig. 1); t, time; σ, sampling; B, distance between turns of the spiral; L, length of spiral; B_0, column perimeter; h, column height; φ, angle formed by the spiral with the vertical; j*, flux; α, thermodiffusional constant; η, D, β, dynamic viscosity, diffusion coefficient, and thermal-expansion coefficient; ρ, density; δ, gap or thickness of wire forming spiral winding; $\Delta T = T_2 - T_1$, $\overline{T} = (1/2)(T_2 + T_1)$; T_2, T_1, temperatures of hot and cold surfaces.

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